Coordinate free analysis of trends in British social mobility

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Abstract

The paper is intended to make a contribution to the ongoing debate about declining social mobility in Great Britain by analyzing mobility tables based on data from the 1991 British Household Panel Survey and the 2005 General Household Survey. The models proposed here generalize Hauser's levels models and allow for semi-parametric analysis of change in social mobility. The cell frequencies are assumed to be equal to the product of three effects: the effect of the Father's position for the given year, the effect of the Son's position for the given year, and the mobility effect related to the difference between the Father's and Son's positions. A generalization of the iterative proportional fitting procedure is proposed and applied to computing the maximum likelihood estimates of the cell frequencies. The standard errors of the estimated parameters are computed under the product multinomial sampling assumption. The results indicate opposing trends of mobility between the two timepoints. Fewer steps up or down in the society became less likely, while more steps became somewhat more likely.

KEY WORDS: exponential family, maximum likelihood estimate, multiplicative model, product multinomial sampling, social mobility

Introduction

A social mobility table is a cross-classification of individuals according to their own and their fathers' social status. If the statuses are ordered, every individual is identified as upward (downward) mobile, i.e., having a higher (lower) status than his father, or immobile, i.e., retaining his father's status. Since greater social mobility is usually considered advantageous for a society, results of analyses comparing the patterns of social mobility across years or between different countries are often given political interpretation. This paper intends to contribute to the debate about declining social mobility in Great Britain [cf. Saunders, 2010].

A social mobility table does not include a variable characterizing mobility per se, and mobility trends are usually inferred from the marginals and the association structure of the table. The marginal distributions reveal changes in the social structure that occurred between the fathers' and sons' generations, and part of mobility can be attributed to those changes. The association structure of the table, operationalized as various sets of odds ratios, reflects the chances of individuals who come from different positions to end up in different social statuses. A weaker association between the fathers' and the sons' social position, called higher social fluidity, implies higher chances of mobility. Although much of social mobility research is concerned with the analysis of the association structure of social mobility tables [cf. Goodman & Hout, 1998, Breen, 2008], neither the marginals nor the association structure quantify social mobility itself, and the question, where is social mobility in the mobility table, persists.

The methods to be described in this paper are based on the observation that social mobility is a common characteristic of some of the cells of the mobility table. For example, if mobility is only considered as being upward or downward, then, if statuses are ordered from highest to lowest, upward mobility is a common characteristic of cells (and observations) in the lower diagonal triangle, downward mobility is a common characteristic of the upper diagonal triangle, and the cells on the main diagonal are characterized by immobility. If social status is considered to have many categories, then mobility can be categorized by the number of steps up or down from the father's position, forming mobility bands parallel to the main diagonal, and the models considered in this paper include effects associated with these bands of mobility. The mobility effects are not based on any of the variables forming the table, and thus these models are coordinate free.

The paper is organized as follows. In Section 1, the structure of a social mobility table, as well as the concepts of absolute and relative mobility are explained in more detail. The most frequently used models for social mobility, including the model of common social fluidity and the UNIDIFF model, are reviewed.

The data used for the analysis in this paper are described in Section 2. These father's position by son's position by year mobility tables were also analyzed by Li & Devine [2011] and are based on the British Household Panel Survey (1991) and on the General Household Survey (2005). Previously, Goldthorpe & Mills [2008] analyzed these data among other data sets in a larger study of trends in social mobility. Some of the results reported in the two papers are different due to somewhat different derivation procedures of the mobility tables. For men, Li & Devine [2011] found a significant decrease in upward mobility and a significant increase in downward mobility from 1991 to 2005, while Goldthorpe & Mills [2008] arrived to almost identical rates of the upward mobility in 1991 and in 2005. However, based on the estimates under the UNIDIFF model, both papers concluded that social fluidity for men was increasing.

The coordinate free relational models [Klimova, Rudas, & Dobra, 2012] that are proposed for the analysis of social mobility tables are given in Section 3. Under these models, the cell frequencies are equal to the product of the effects that exist under conditional independence of respondents' and their fathers' positions given year, that is the father by year and son by year effects, and the effects associated with the subsets of cells exhibiting the same level of social mobility. Mobility is measured by the number and direction of steps from the origin; the diagonal (immobile) cells are unconstrained by these models. The model assuming that social mobility changed between 1991 and 2005 is tested against a more restrictive model assuming no change. The maximum likelihood estimates of the cell frequencies are computed using an algorithm that generalizes the traditional iterative proportional fitting (IPF) procedure and is given Section 3. The generalization proposed here adjusts the sums of the cell frequencies for subsets of cells that exhibit a common effect in the model to their observed values. Finally, the standard errors of the estimated parameters are computed under the product-multinomial sampling assumption using the approach of Lang [1996].

The results of the analyses are discussed in Section 4. The model of no change does not fit well, and thus the data provide some evidence that mobility in Great Britain changed between the two timepoints. The model allowing change in mobility, but assuming that the mobility effect, in each year, depends only on the number and direction of steps taken by an individual, fits well. The parameter estimates for mobility effects obtained from this model indicate opposing trends of mobility between 1991 and 2005. In 2005, fewer steps up or down in the society became less likely than in 1991, while more steps became more likely. These results provide a finer picture of the patterns of change in social mobility in contemporary Great Britain than earlier analyses did.

1 Models of Social Mobility

If a society is stratified according to some criteria, then every individual can be assigned to one and only one stratum. The strata are often referred to as social classes or statuses. Let I be the total number of social classes in the stratification of interest. A social mobility table is a cross-classification of two variables - Origin, the position in the society from which an individual came, and Destination, the position which the individual is now having. Origin usually refers to the Father's status (e.g. Father's job, when the respondent was at the age 16), and Destination refers to Son's status (e.g. respondent's current full-time job).

A mobility table can be characterized by rates of absolute mobility and by rates of relative mobility. Absolute mobility comprises the upward mobility and downward mobility and is expressed as the percentage of the respondents who moved, respectively, up or down compared to their origin. Relative mobility is described by the association structure of the table and is expressed in terms of the odds ratios

$$\frac{p_{ii}/p_{ij}}{p_{ji}/p_{jj}}, \quad i, j = 1, \ldots, I.$$

Thus, relative rates indicate the chances of an individual from the class i to stay in this class rather than move to the class j, relative to the chances of an individual from the class j to move to the class i rather than stay in class j [cf. Goldthorpe & Jackson, 2007].

Perfect mobility means independence between Father's status and Son's status, and under this model, the log cell probabilities in the mobility table can be written in the log-linear representation of

$$\log p_{ij} = \lambda + \lambda_i^F + \lambda_j^S, \quad i, j = 1, \ldots, I, \tag{1}$$

where λ_i^F and λ_j^S are effects associated with Father's status and with Son's status, respectively. This model does not fit most data sets, because of too many immobile observations, that is sons who retained their fathers' positions. The model of quasi-independence [Goodman, 1968] entirely removes restrictions from the diagonal cells by allowing for additional parameters:

$$\log p_{ij} = \lambda + \lambda_i^F + \lambda_j^S + \delta_i \mathbf{I}(i = j), \quad i, j = 1, \ldots, I, \tag{2}$$

where

$$\mathbf{I}(i = j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases},$$

and the parameters δ_i characterize the departure of the diagonal cells from independence.

Under the levels model [Hauser, 1978], also called topological model [Hout, 1983], the log cell probabilities are sums of the effects associated with Father's status, Son's status and some interaction effects, that are assumed to be identical for all cells that belong to the same level. The levels are determined empirically from the ratios of the frequencies expected under the model of independence to those observed. Clogg & Shockey [1984] suggested that cells belonging to the same level have to share some common substantive property. In turn, Kovách, Róbert, & Rudas [1986] considered levels consisting of contiguous cells. Models which accounted for the social distance between Father's and Son's statuses, as if it was measured on the interval scale, were proposed by, e.g., Goodman [1972], Sobel [1981], Hope [1982], Agresti [1983], Hauser [1984]. These models assume, for social statuses numbered from 1 to I, that the distance between categories i_1 and i_2 is the same as between categories i_3 and i_4 ($1 \leq i_1, i_2, i_3, i_4 \leq I$), if $|i_1 - i_2| = |i_3 - i_4|$.

Comparative mobility research concentrates on revealing changes in social mobility across years or between countries and is based on analyzing the relative mobility rates, or patterns of the association between Father's and Son's status in several social mobility tables.

The model of conditional independence of Father's and Son's status given Year (Y) assumes that there is no association between Father's and Son's status within each year:

$$\log p_{ijk} = \lambda + \lambda_i^F + \lambda_j^S + \lambda_k^Y + \lambda_{ik}^{FY} + \lambda_{jk}^{SY}, \;\; i, j = 1, \ldots, I, \; k = 1, \ldots, K. \tag{3}$$

Under the model of common (or constant) social fluidity (CSF), see [Erikson & Goldthorpe, 1992], the association between Father's and Son's status is allowed to exist and be different in every cell, but is assumed to be identical for each year (no second order association between F, S, and Y):

$$\log p_{ijk} = \lambda + \lambda_i^F + \lambda_j^S + \lambda_k^Y + \lambda_{ik}^{FY} + \lambda_{jk}^{SY} + \lambda_{ij}^{FS}, \;\; i, j = 1, \ldots, I, \; k = 1, \ldots, K. \tag{4}$$

If, instead of the λ_{ij}^{FS} term, Hauser's levels are allowed, the model of constant pattern of fluidity [Erikson et al., 1982] is obtained.

A generalization of this model is the UNIDIFF (uniform difference) model [Erikson & Goldthorpe, 1992], also called the log-multiplicative layer effect model [Xie, 1992]:

$$\log p_{ijk} = \lambda + \lambda_i^F + \lambda_j^S + \lambda_k^Y + \lambda_{ik}^{FY} + \lambda_{jk}^{SY} + \psi_{ij}\phi_k, \;\; i, j = 1, \ldots, I, \; k = 1, \ldots, K. \tag{5}$$

Under this model, the $F \times S \times Y$ association is separated into the $F \times S$ and Y components. The pattern of the $F \times S$ association, expressed by the parameters ψ_{ij}, is assumed to be the same across years, but the relative strength of this association, expressed by the parameters ϕ_k, can differ for $k = 1, \ldots, K$. Depending on the constraints placed on the parameters ψ_{ij} and ϕ_k, this model can be just a reparameterization of the CSF model or can lead to some other models for mobility tables [Xie, 1992]. A UNIDIFF model is used to test whether the odds ratios in the kth year are uniformly higher or uniformly lower than the odds ratios in the reference year. However, there are cases when the pattern of mobility does not change as specified by a UNIDIFF model [cf. Wong, 1994].

This paper offers an approach that is based on the relational model framework, recently proposed by Klimova et al. [2012]. This approach is a generalization of the levels model framework [Hauser, 1978], is not data driven, and allows for semi-parametric modeling of change in the mobility pattern. Under the relational models of mobility described in this paper (see Section 3), the cell frequencies arise as the products of effects associated with the marginal distributions and also with subsets of cells exhibiting the same level of social mobility.

2 Data and Previous Analyses

The analysis in this paper is based on the two data sets that were also analyzed in "Trends in intergenerational class mobility in modern Britain: evidence from national surveys, 1972–2005" [Goldthorpe & Mills, 2008], referred to as TICM in the sequel, and "Is social mobility really declining?" [Li & Devine, 2011], referred to as SMD. Both TICM and SMD relied on the British Household Panel Survey (1991) and the General Household Survey (2005). The mobility tables used by TICM and SMD are based on seven categories of class positions: 1. Higher managerial and professional; 2. Lower managerial and professional; 3. Intermediate occupations (clerical, sales, service); 4. Small employers and own account workers; 5. Lower supervisory and technical; 6. Semi-routine; 7. Routine. Although the authors of SDM aimed to reproduce and expand the analyses described in TICM for the two surveys, they used somewhat different procedures to derive the occupational statuses from the survey data. Consequently, the results in TICM and SDM are not the same. For example, while TICM reports almost identical rates of the upward mobility for men in 1991 and 2005, SDM detects a significant decrease in upward mobility and a significant increase in downward mobility for men. The comparison of the fit statistics obtained in these papers is given in Table 1. Despite the differences, both papers conclude that association between Father's and Son's positions was decreasing, that is social fluidity for men was increasing.

The percentage data obtained in SDM for male respondents of age 25-59, transformed to frequencies and given in Table 2, are used for the analysis in this paper.

3 Relational Models of Social Mobility

Let $\mathbb{T}$ be a contingency table. In the population, each cell $t \in \mathbb{T}$ has a fixed probability $p(t) \in (0, 1)$, such that $\sum_{t=1}^{T} p(t) = 1$, where T stands for the number of cells in the table.

Table 1: **Fit statistics and p-values**

Model	df	Goldthorpe & Mills [2008]		Li & Devine [2011]	
		G^2	p-value	G^2	p-value
Cond Ind	72	756.9	0.00	652.8	0.00
CSF	36	19.3	0.99	51.3	0.00
UNIDIFF	35	15.5	0.99	34.5	0.49

Let $\mathcal{P}$ denote the set of such probability distributions. Let $\mathbf{U} = \{U_1, \dots, U_S\}$ be a class of non-empty subsets of the table $\mathbb{T}$ and $\mathbf{A}$ be a $S \times T$ matrix with entries

$$a_{st} = \mathbf{I}_s(t) = \begin{cases} 1, & \text{if the } t\text{-th cell is in } U_s, \\ 0, & \text{otherwise,} \end{cases} \quad \text{for } t = 1, \dots, T \text{ and } s = 1, \dots, S.$$

A relational model $RM(\mathbf{U})$ [Klimova et al., 2012] with the model matrix $\mathbf{A}$ is the following subset of $\mathcal{P}$:

$$RM(\mathbf{U}) = \{\boldsymbol{p} \in \mathcal{P}: \ \log \boldsymbol{p} = \mathbf{A}'\boldsymbol{\beta}, \text{ for some } \boldsymbol{\beta} \in \mathbb{R}^S\}.$$

Without loss of generality, it is assumed that the model matrix $\mathbf{A}$ is of full row rank.

The kernel of the model matrix $Ker(\mathbf{A}) = \{\boldsymbol{\gamma} \in \mathbb{R}^T: \ \mathbf{A}\boldsymbol{\gamma} = \mathbf{0}\}$ is a linear subspace. A matrix $\mathbf{D}$ with rows that form a basis of $Ker(\mathbf{A})$ is called a kernel basis matrix of $RM(\mathbf{U})$ and can be chosen to have integer entries [Klimova et al., 2012, Theorem 2.2]. The distribution $\boldsymbol{p}$ belongs to the relational model $RM(\mathbf{U})$ if and only if

$$\mathbf{D}\log \boldsymbol{p} = \mathbf{0}.$$

Relational models generalize, among others, the quasi models [Goodman, 1968, 1972], topological models [Hauser, 1978, Hout, 1983], indicator models [Zelterman & Youn, 1992].

In the remainder of this section, two relational models for the comparative analysis of mobility tables will be described. The model (3) of conditional independence of Father's and Son's status given Year will be used as the baseline model. This model assumes that, within a given year, Father's and Son's effects are independent from each other and thus the relative rates of mobility are equal to 1. Since this model doesn't fit the data (see Table 1), further, mobility-related, effects are included.

The relational models will be generated by the cylinder subsets associated with the $F \times Y$ effect and $S \times Y$ effect and additional subsets, called mobility bands. A mobility band is a collection of those cells that represent the same number and direction of steps from Father's position to Son's position within each year. Each diagonal cell has its own effect, so that

Table 2: **Distribution of men by class of origin and destination**

1991		Destination 1	2	3	4	5	6	7
Origin	1	50	47	5	11	11	5	8
	2	124	166	24	61	55	29	47
	3	37	50	8	18	24	13	21
	4	61	66	13	97	50	21	53
	5	71	113	24	74	100	74	103
	6	34	71	11	50	69	74	95
	7	40	79	18	71	105	74	105
2005		**Destination 1**	**2**	**3**	**4**	**5**	**6**	**7**
Origin	1	182	139	28	32	28	24	48
	2	246	297	51	123	91	87	127
	3	67	95	12	36	40	24	40
	4	55	79	12	63	55	51	59
	5	99	139	28	75	87	75	103
	6	75	115	12	75	95	91	119
	7	67	119	16	79	91	79	135

model fit is not influenced by immobile cells. The definition of mobility related subsets U_h is given in Table 3, where cells belonging to the same subset are marked with the same number. Under this model, called M_{diff},

$$\log p_{ijk} = \lambda_0 + \lambda_i^F + \lambda_j^S + \lambda_k^Y + \lambda_{ik}^{FY} + \lambda_{jk}^{SY} + \sum_{h=1}^{38} \beta_h \mathbf{I}_{U_h}(i,j,k),$$

for $i, j = 1, \ldots, 7$, $k = 1, 2$. Here $\mathbf{I}_{U_h}$, for each $h = 1, \ldots, 38$, is the indicator function of the mobility band U_h.

Under the model M_{diff}, the mobility effects are allowed to be different for the two years. The fit of this model will be compared with that of the model M_{same}. Under the model M_{same}, the mobility effects are supposed to be the same for the two years. The definition of the mobility-related subsets V_h is given in Table 4, and the model can be written as

$$\log p_{ijk} = \lambda_0 + \lambda_i^F + \lambda_j^S + \lambda_k^Y + \lambda_{ik}^{FY} + \lambda_{jk}^{SY} + \sum_{h=1}^{26} \gamma_h \mathbf{I}_{V_h}(i,j,k),$$

for $i, j = 1, \ldots, 7$, $k = 1, 2$. Here $\mathbf{I}_{V_h}$, for each $h = 1, \ldots, 26$, is the indicator function of the mobility band V_h.

Table 3: **Mobility bands U_h in model M_{diff}**

1991		Destination 1	2	3	4	5	6	7
Origin	1	13	1	2	3	4	5	6
	2	7	14	1	2	3	4	5
	3	8	7	15	1	2	3	4
	4	9	8	7	16	1	2	3
	5	10	9	8	7	17	1	2
	6	11	10	9	8	7	18	1
	7	12	11	10	9	8	7	19
2005		**Destination 1**	**2**	**3**	**4**	**5**	**6**	**7**
Origin	1	32	20	21	22	23	24	25
	2	26	33	20	21	22	23	24
	3	27	26	34	20	21	22	23
	4	28	27	26	35	20	21	22
	5	29	28	27	26	36	20	21
	6	30	29	28	27	26	37	20
	7	31	30	29	28	27	26	38

Table 4: **Mobility bands V_h in model M_{same}**

1991		Destination 1	2	3	4	5	6	7
Origin	1	13	1	2	3	4	5	6
	2	7	14	1	2	3	4	5
	3	8	7	15	1	2	3	4
	4	9	8	7	16	1	2	3
	5	10	9	8	7	17	1	2
	6	11	10	9	8	7	18	1
	7	12	11	10	9	8	7	19
2005		**Destination 1**	**2**	**3**	**4**	**5**	**6**	**7**
Origin	1	20	1	2	3	4	5	6
	2	7	21	1	2	3	4	5
	3	8	7	22	1	2	3	4
	4	9	8	7	23	1	2	3
	5	10	9	8	7	24	1	2
	6	11	10	9	8	7	25	1
	7	12	11	10	9	8	7	26

The usual restrictions imposed on the parameters related to conditional independence

$$\begin{aligned}
\sum_{i=1}^{7} \lambda_i^F &= 0, \ \sum_{j=1}^{7} \lambda_j^S = 0, \ \sum_{k=1}^{2} \lambda_k^Y = 0, \\
\sum_{k=1}^{2} \lambda_{ik}^{FY} &= 0, \ \sum_{k=1}^{2} \lambda_{jk}^{SY} = 0, \quad \text{for } i, j = 1, \ldots, 7; \\
\sum_{i=1}^{7} \lambda_{ik}^{FY} &= 0, \ \sum_{j=1}^{7} \lambda_{jk}^{SY} = 0, \quad \text{for } k = 1, 2,
\end{aligned}$$

do not suffice to make all parameters of the models identifiable, and more constraints are needed. The band parameters associated with upward and with downward mobility are thus centered around 1 on the multiplicative scale within both years for the model M_{diff}:

$$\sum_{h=1}^{6} \beta_h = 0, \ \sum_{h=7}^{12} \beta_h = 0, \ \sum_{h=20}^{25} \beta_h = 0, \ \sum_{h=26}^{31} \beta_h = 0, \tag{6}$$

and are centered around 1 on the multiplicative scale for the model M_{same}:

$$\sum_{h=1}^{6} \gamma_h = 0, \ \sum_{h=7}^{12} \gamma_h = 0.$$

Both models include the overall effect, expressed by the parameter λ_0, and hence are regular exponential families [Klimova et al., 2012, Theorem 3.2]. The maximum likelihood (ML) estimates of cell frequencies under these models exist and are unique [cf. Barndorff-Nielsen, 1978].

The ML estimates of the cell frequencies can be obtained using an iterative proportional fitting (IPF) procedure that starts with a distribution in the model and adjusts the subset sums to their observed values. Let $RM(\mathbf{U})$ be a relational model with the overall effect, $\boldsymbol{y} = \{y(t)\}$ be the observed frequency distribution, and $\hat{\boldsymbol{y}} = \{\hat{y}(t)\}$ denote the MLE of $\boldsymbol{y}$ under the model.

Algorithm:

Set $m^0(t) = 1$ for all $t = 1, \ldots, T$. Cycle for $d = 0, 1, 2, \ldots$:

1. $U_{d+1} = U_s$, if $d + 1 \equiv s \bmod S$, $\quad 1 \leq s \leq S$.

2. for all $t \in \mathbb{T}$

$$m^{d+1}(t) = \begin{cases} m^d(t) \frac{\sum_{r \in \mathbb{T}} \mathbf{I}_{U_{d+1}}(r) y(r)}{\sum_{r \in \mathbb{T}} \mathbf{I}_{U_{d+1}}(r) m^d(r)} & \text{if } t \in U_{d+1}, \\ m^d(t) & \text{otherwise.} \end{cases}$$

This procedure is a generalization of the standard IPF algorithm [cf. Bishop, Fienberg, & Holland, 1975], but the proof of convergence given by Csiszár [1975] applies in this case too, and the sequence $\boldsymbol{m}^d$ converges, as $d \to \infty$, to $\hat{\boldsymbol{y}}$.

The data used in this analysis were assembled from two independent surveys, and thus product-multinomial sampling is a better approximation of reality than multinomial sampling. Asymptotic standard errors for log-linear models under the product-multinomial sampling scheme were obtained by Lang [1996]. Although the relational models for mobility tables described in this paper are more general than log-linear models, the derivation techniques originally proposed by Aitchinson & Silvey [1958] apply. If $\mathbf{A}$ and $\mathbf{D}$ stand, respectively, for the model matrix and a kernel basis matrix of M_{diff}, then the estimate of the covariance matrix of the coefficients of the model is equal to [Lang, 1996, Eq.(4.5)]

$$\hat{\text{Cov}}(\hat{\boldsymbol{\beta}}) = (\mathbf{A}\mathbf{A}')^{-1}\mathbf{A}\hat{\text{Cov}}(\hat{\boldsymbol{f}})\mathbf{A}'(\mathbf{A}\mathbf{A}')^{-1}. \tag{7}$$

The estimate of the covariance matrix of the cell frequencies equals [Lang, 1996, Eq.(4.6)]

$$\hat{\text{Cov}}(\hat{\boldsymbol{f}}) = (\text{diag}(\hat{\boldsymbol{f}})) - \mathbf{D}'(\mathbf{D}(\text{diag}(\hat{\boldsymbol{f}}))^{-1}\mathbf{D}')^{-1}\mathbf{D} - \begin{pmatrix} \hat{\boldsymbol{f}}_1\hat{\boldsymbol{f}}_1'/N_1 & \boldsymbol{O} \\ \boldsymbol{O} & \hat{\boldsymbol{f}}_2\hat{\boldsymbol{f}}_2'/N_2 \end{pmatrix} \tag{8}$$

Here $N_1 = 2630$ and $N_2 = 3965$ are the numbers of respondents in 1991 and in 2005, respectively.

4 Results

The fit statistics and p-values for the models M_{same} and M_{diff} are given in Table 5. The model M_{same} does not fit well and, thus, the data provide some evidence that the pattern of mobility changed between the two timepoints. Parameter estimates from the model M_{diff} will be used to describe these changes.

Table 5: **Relational models: fit statistics and p-values**

Model	df	G^2	p-value
M_{diff}	38	45.18	0.19
M_{same}	48	66.94	0.04

Table 6: **Estimated mobility effects (standard errors) for the model M_{diff}**

Distance from the origin	1991	2005	Ratio
6 steps down	0.44 (0.13)	0.78 (0.10)	1.77 (0.56)
5 steps down	0.69 (0.10)	0.79 (0.06)	1.14 (0.19)
4 steps down	0.93 (0.12)	0.86 (0.07)	0.92 (0.14)
3 steps down	1.25 (0.13)	0.93 (0.07)	0.74 (0.09)
2 steps down	1.46 (0.13)	1.35(0.08)	0.92 (0.10)
1 step down	1.92 (0.14)	1.49 (0.08)	0.78 (0.07)
1 step up	1.78 (0.11)	1.45 (0.07)	0.81 (0.06)
2 steps up	1.57 (0.11)	1.30 (0.08)	0.83 (0.07)
3 steps up	1.33 (0.09)	1.10 (0.07)	0.83 (0.08)
4 steps up	1.01 (0.08)	0.97 (0.07)	0.96 (0.10)
5 steps up	0.66 (0.06)	0.82 (0.06)	1.24 (0.15)
6 steps up	0.41 (0.06)	0.60 (0.07)	1.46 (0.27)

The ML estimates, under the model M_{diff}, for the mobility effects (on the multiplicative scale), their ratios, and the applicable standard errors are shown in Table 6 [1]. The effects are centered around 1 on both sides of the main diagonal by assumption (6). The standard errors for the mobility effects were computed from (7), and the standard errors for the ratios were obtained using the delta method.

[1]Parameter estimation for relational models that contain the overall effect, like M_{diff}, can be performed using the *gnm*() function in the R package. However, for the models proposed in this paper, the *gnm*() results have shown some instability, and the authors used their own code, which is available upon request.

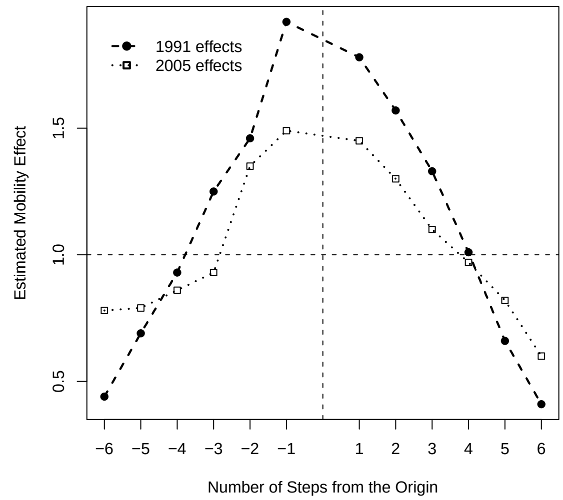

Figure 1: Estimated mobility effects under M_{diff}.

The estimated mobility effects for both years versus the number of steps up or down from the origin are displayed in Figure 1. The plot indicates that mobility decreases monotonically as the number of steps from the origin increases. Therefore, moving a small number of steps up or down compared to the origin seems to be easier than making larger steps. For 1991, the plot is remarkably symmetric (the effects depend very little on direction), less so in 2005. In 2005, small moves (1-3 steps) appear to be less likely than in 1991, and for 5 out of these 6 levels the changes are statistically significant at the 95% level. For 4 steps of mobility, the chances in the two years are virtually identical. Large moves (5-6 steps) appear to be slightly more likely in 2005 than in 1991, but the increase is not statistically significant (see Table 6). However, this lack of significance does not occur because of smaller estimated effect sizes for increased chances of more steps of mobility in 2005 compared to 1991, rather because of larger estimated standard errors for these quantities.

5 Discussion

The analyses based on the relational models described in this paper offer a number of advantages in comparative mobility research. The proposed models allow for mobility effects that are not associated with variables or groups of variables in the table. An $F \times S$ interaction is not assumed beyond the one associated with having made a certain number of steps up or down, and a finer analysis of the changing mobility patterns is possible. The mobility effect in the model M_{diff} is not assumed to be a linear or some other parametric function of the

$F \times S$ interaction and is therefore modeled in a semi-parametric way. This makes it possible to analyze different numbers and directions of steps independently of each other, revealing effects when change is statistically significant for some levels but not significant for others. The results of the paper show that there is no simple "yes" or "no" response to the question whether social mobility declined in Great Britain during the last decades. Further, it has to be kept in mind that the results may depend on the definition of the social statuses used and the categorization of the respondents into these statuses. Using different status definitions or different procedures to assign respondents to these statuses may lead to different findings.

Relational models retain the flexibility of Hauser's topological models, and the coordinate free nature makes it simpler to add additional attributes to the analyses, e.g., grandparents' or siblings' social statuses. Finally, the semi-parametric approach, based on the relational models and demonstrated in the paper, can be applied to modeling structure or change in structure in other areas, including biology or social networks, see [Klimova et al., 2012].

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References

Agresti, A. (1983). A simple diagonals-parameter symmetry and quasi-symmetry model. *Statist. Probab. Lett.*, *1*, 313–316.

Aitchinson, J., & Silvey, S. D. (1958). Maximum-likelihood estimation of parameters subject to restraints. *The Annals of Mathematical Statistics*, *29*, 813–828.

Barndorff-Nielsen, O. E. (1978). *Information and exponential families.* New York: Wiley.

Bishop, Y. M. M., Fienberg, S. E., & Holland, P. W. (1975). *Discrete multivariate analysis: Theory and practice.* MIT.

Breen, R. (2008). Statistical models of association for comparing cross-classification. *Sociol.Methods Res.*, *36*, 442–461.

Clogg, C. C., & Shockey, J. W. (1984). A note on two models for mobility tables. *Comparative Social Research*, *7*, 443–462.

Csiszár, I. (1975). I-divergence geometry of probability distributions and minimization problems. *Ann. Probab.*, *3*, 146–158.

Erikson, R., & Goldthorpe, J. H. (1992). *The constant flux*. Clarendon Press, Oxford.

Erikson, R., Goldthorpe, J. H., & Portocarero, L. (1982). Social fluidity in industrial nations: England, France and Sweden. *The British Journal of Sociology*, *33*, 1–34.

Goldthorpe, J. H., & Jackson, M. (2007). Intergenerational class mobility in contemporary Britain: political concerns and empirical findings. *British Journal of Sociology*, *58*, 525–546.

Goldthorpe, J. H., & Mills, C. (2008). Trends in intergenerational class mobility in modern Britain: evidence from national surveys, 1972–2005. *National Institute Economic Review*, *205*, 83–100.

Goodman, L. A. (1968). The analysis of cross-classified data: independence, quasi-independence, and interaction in contingency tables with or without missing cells. *J. Amer. Statist. Assoc.*, *63*, 1091–1131.

Goodman, L. A. (1972). Some multiplicative models for the analysis of cross-classified data. *Proceedings of the Sixth Berkley Symposium on Mathematical Statistics and Probability*.

Goodman, L. A., & Hout, M. (1998). Statistical methods and graphical displays for analyzing how the association between two qualitative variables differs among countries, among groups, or over time: A modified regression-type approach. *Sociological Methodology*, *28*, 175–230.

Hauser, R. M. (1978). A structural model of the mobility table. *Social Forces*, *56*, 919–953.

Hauser, R. M. (1984). Vertical class mobility in England, France, and Sweden. *Acta Sociologica*, *27*, 87–110.

Hope, K. (1982). Vertical and nonvertical class mobility in three countries. *American Sociological Review*, *47*, 99–113.

Hout, M. (1983). *Mobility tables* (Vol. 31). Sage Publications, Inc.

Klimova, A., Rudas, T., & Dobra, A. (2012). Relational models for contingency tables. *J. Multivariate Anal.*, *104*, 159–173.

Kovách, I., Róbert, P., & Rudas, T. (1986). Towards the dimensions of mobility. *in Andorka, R., Bertalan, L.(eds.) Economy and Society in Hungary, Budapest*, 153–183.

Lang, J. B. (1996). On the comparison of multinomial and Poisson log-linear models. *J. Roy. Statist. Soc. Ser.B*, *58*, 253–266.

Li, Y., & Devine, F. (2011). Is social mobility really declining? *Sociological Research Online*, *16*.

Saunders, P. (2010). *Social mobility myths.* London: Civitas.

Sobel, M. E. (1981). Diagonal mobility models: A substantively motivated class of designs for the analysis of mobility effects. *American Sociological Review*, *46*, 893–906.

Wong, R. S. (1994). Postwar mobility trends in advanced industrial societies. *Research in Social Stratification and Mobility*, *13*, 121–144.

Xie, Y. (1992). The log-multiplicative layer effect model for comparing mobility tables. *American Sociological Review*, *57*, 380–395.

Zelterman, D., & Youn, T. I. (1992). Indicator models for social mobility tables. *Comput. Statist. Data Anal.*, *14*.